Homework 1, Morally Due Tue Feb 6, 2018

1. (5 points) What is your name? Write it clearly. Staple your HW. When is the midterm tentatively scheduled (give Date and Time)? If you cannot make it in that day/time see me ASAP.

2. (25 points)
   (a) (10 points) Prove that for every \( c \), for every \( c \) coloring of \( \binom{N}{2} \), there is a homogenous set USING a proof similar to what I did in class.
   (b) (10 points) Prove that for every \( c \), for every \( c \) coloring of \( \binom{N}{2} \), there is an infinite homogenous set USING induction on \( c \).
   (c) (0 points) Which proof do you like better? Which one do you think gives better bound when you finitize it?

3. (20 points) State and prove (rigorously) the \( c \)-color \( a \)-ary Ramsey Theorem. Your statement should start out for all \( a \geq 1 \), for all \( c \geq 1 \), .... The proof should be by induction on \( a \) with the base case being \( a = 1 \).

4. (25 points)
   (a) Look up a proof of the Bolzano-Wierstrauss Theorem and present it in your own words.
   (b) THINK ABOUT: Is it similar to the proof of Ramsey’s theorem?
   (c) LISTEN TO the one of the many rap songs about the BW theorem:
       www.youtube.com/watch?v=dfO18klwKHg
       (There is also a link on the website.)
       What did you think of it?

5. (25 points) State and prove a theorem with the XXX filled in.
   For every coloring (any number of colors) of XXX\( (n) \) points there is EITHER: (a) a set of \( n \) that are all colored the same, or (b) a set of \( n \) points that are all colored differently. However!- there IS a coloring of XXX\( (n) \) – 1 points such that there is NEITHER: (a) a set of \( n \) that are all colored the same, or (b) a set of \( n \) points that are all colored differently.