HW 12 CMSC 456. MORALLY DUE Dec 10
SOLUTIONS
NOTE- THE HW IS FOUR PAGES LONG

1. (0 points) READ the syllabus- Content and Policy. What is your name? Write it clearly. What is the day of the final? READ the slides and notes on Perfect and Comp Secrecy.

GOTO NEXT PAGE
2. (40 points)

All of the arithmetic in this problem is mod 2.

Write a program do do the following.

Input \(c_0, c_1, c_2, c_3 \in \{0, 1\}\) and do the following:

(a) Let

\[
\begin{align*}
\ f(s_3, s_2, s_1, s_0) &= (c_3 s_3 + c_2 s_2 + c_1 s_1 + c_0 s_0, s_3, s_2, s_1) \\
\end{align*}
\]

(ADDED: This is CORRECT. Earlier version had

\[
\begin{align*}
\ (c_3 s_3 + c_2 s_2 + c_1 s_1 + c_0 s_1, s_3, s_2, s_1) \\
\end{align*}
\]

which was INCORRECT.)

(b) For all \(b_0, b_1, b_2, b_3 \in \{0, 1\}\) compute

\[
\begin{align*}
\ v_0 &= (b_3, b_2, b_1, b_0) \\
\ v_1 &= f(v_0) \\
\ v_2 &= f(v_1) \\
\vdots \\
\ \text{UNTIL you find } i < j \text{ such that } f(v_i) = f(v_j). \\
\end{align*}
\]

Keep track of the \(j\)'s seen (see next step). (So you will need to store all of the \(v_1, v_2, \ldots\). Since there are only 16 possibilities you can do this in an inelegant but easy way. And DO NOT WORRY- I am NOT going to ask you to later to it for 10 bits or 100 bits or something that would require a clever way to store it.)

(c) For each \((c_3, c_2, c_1, c_0)\) note which \((b_3, b_2, b_1, b_0)\) lead to the LARGEST sequence without a repeat- so the largest \(j\).

You final output should look like this (I made up the numbers and only give the first two rows. Yours should have 16 rows).

<table>
<thead>
<tr>
<th>c-vector</th>
<th>best b-vector</th>
<th>length of sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
<td>1100</td>
<td>2</td>
</tr>
<tr>
<td>0001</td>
<td>1010</td>
<td>13</td>
</tr>
</tbody>
</table>

SOLUTION TO PROBLEM TWO

Omitted.

END OF SOLUTION TO PROBLEM TWO

GOTO NEXT PAGE
3. (30 points) All of the arithmetic in this problem is mod 2.

Let

\[ f(s_3, s_2, s_1, s_0) = (s_3 s_2 + s_1 + s_0, s_3, s_2, s_1) \]

(ADDED: No problem here. The above IS correct and always has been. Some students THOUGHT it was a typo to have \( s_3 s_2 \) but its NOT. This is close to the function I proposed on Nov 19, slide titled Nonlinear Feedback Shift Register)

Write a program do do the following.

(a) For all \( b_0, b_1, b_2, b_3 \) compute

\[ v_0 = (b_3, b_2, b_1, b_0) \]
\[ v_1 = f(v_0) \]
\[ v_2 = f(v_1) \]

\[ \vdots \]

UNTIL you find \( i < j \) such that \( f(v_i) = f(v_j) \). Keep track of the \( j \)'s seen (see next step). (So you will need to store all of the \( v_1, v_2, \ldots \). Since there are only 16 possibilities you can do this in an inelegant but easy way. And DO NOT WORRY- I am NOT going to ask you to later to it for 10 bits or 100 bits or something that would require a clever way to store it.)

(b) For each \( (b_3, b_2, b_1, b_0) \) output the length of the sequence before a repeat. sequence without a repeat.

You final output should look like this (I made up the numbers and only gave the first two rows. Yours should have 16 rows.)

<table>
<thead>
<tr>
<th>b-vector</th>
<th>length of sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
<td>5</td>
</tr>
<tr>
<td>0001</td>
<td>19</td>
</tr>
</tbody>
</table>

SOLUTION TO PROBLEM THREE

Omitted

END OF SOLUTION TO PROBLEM THREE

GOTO NEXT PAGE
4. (30 points) Give a rigorous definition of a pseudorandom FUNCTION that uses a game. It should begin with $F_k(x)$ where $k$ is unif in $\{0,1\}^n$. $F_k$ goes from $\{0,1\}^n$ to $\{0,1\}^n$.

(ADDED HINT:

- Use a Game!
- In class we have defined roughly two kinds of games: Here is an example of each: (1) Defining perfect security. Here Eve picks $m_0, m_1$, Alice encodes one of them into $c$, gives Eve $c$, and eve has to tell which one it was $m_0$ or $m_1$. (2) Defining pseudorandom GEN: Here ALICE picks a truly random string and a pseudorandom string and Eve has, gives one of them to Eve, Eve has to tell which one it was.
- The definition of pseudorandom Function will be more like that of pseudorandom generator.
- When Alice gives Eve a function, Eve will have black box access to it.

) SOLUTION TO PROBLEM FOUR

We define the following game:

(a) Alice picks $k \in \{0,1\}^n$. (She is really picking $F_k$.) Let $f_1$ be $F_k$.
(b) Alice picks a function unif at random that goes from $\{0,1\}^n$ to $\{0,1\}^n$. Let this function be $f_2$.
(c) Alice lets $G_1$ be one of $\{f_1, f_2\}$ and $G_2$ be the other. This is determined by coin flip.
(d) Eve gets black box access to $G_1$ and to $G_2$ but is not told which is which. Hence Eve can make queries to either one, many of them (we will later discuss Eve’s computational limits.)
(e) Eve asserts which of $G_1$ and $G_2$ is $f_1$. If she gets it right she wins!

$F_k$ is a PseudoRandom function if for all PPT Eve, for all neg $\epsilon(n)$, the prob that Eve wins is $\leq \frac{1}{2} + \epsilon(n)$.

END OF SOLUTION TO PROBLEM FOUR