HW 12 CMSC 456. MORALLY DUE Dec 10
NOTE- THE HW IS FOUR PAGES LONG

1. (0 points) READ the syllabus- Content and Policy. What is your name? Write it clearly. What is the day of the final? READ the slides and notes on Perfect and Comp Secrecy.

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2. (40 points)

All of the arithmetic in this problem is mod 2.
Write a program to do the following.
Input \(c_0, c_1, c_2, c_3 \in \{0, 1\}\) and do the following:

(a) Let

\[
  f(s_3, s_2, s_1, s_0) = (c_3s_3 + c_2s_2 + c_1s_1 + c_0s_0, s_3, s_2, s_1)
\]

(ADDED: This is CORRECT. Earlier version had

\[
  (c_3s_3 + c_2s_2 + c_1s_1 + c_0s_0, s_3, s_2, s_1)
\]

which was INCORRECT.)

(b) For all \(b_0, b_1, b_2, b_3 \in \{0, 1\}\) compute

\[
  v_0 = (b_3, b_2, b_1, b_0) \\
  v_1 = f(v_0) \\
  v_2 = f(v_1) \\
  \vdots
\]

UNTIL you find \(i < j\) such that \(f(v_i) = f(v_j)\). Keep track of the \(j\)'s seen (see next step). (So you will need to store all of the \(v_1, v_2, \ldots\). Since there are only 16 possibilities you can do this in an inelegant but easy way. And DO NOT WORRY- I am NOT going to ask you to later to it for 10 bits or 100 bits or something that would require a clever way to store it.)

(c) For each \((c_3, c_2, c_1, c_0)\) note which \((b_3, b_2, b_1, b_0)\) lead to the LARGEST sequence without a repeat- so the largest \(j\).

You final output should look like this (I made up the numbers and only give the first two rows. Yours should have 16 rows).

<table>
<thead>
<tr>
<th>(c)-vector</th>
<th>best (b)-vector</th>
<th>length of sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
<td>1100</td>
<td>2</td>
</tr>
<tr>
<td>0001</td>
<td>1010</td>
<td>13</td>
</tr>
</tbody>
</table>
3. (30 points) All of the arithmetic in this problem is mod 2.

Let

\[ f(s_3, s_2, s_1, s_0) = (s_3s_2 + s_1 + s_0, s_3, s_2, s_1) \]

(ADDED: No problem here. The above IS correct and always has been. Some students THOUGHT it was a typo to have \( s_3s_2 \) but its NOT. This is close to the function I proposed on Nov 19, slide titled \textit{Nonlinear Feedback Shift Register} )

Write a program do do the following.

(a) For all \( b_0, b_1, b_2, b_3 \) compute

\[
\begin{align*}
 v_0 &= (b_3, b_2, b_1, b_0) \\
 v_1 &= f(v_0) \\
 v_2 &= f(v_1) \\
 &\ldots \\
 \text{UNTIL you find } i < j \text{ such that } f(v_i) = f(v_j). \text{ Keep track of the } j \text{'s seen (see next step). (So you will need to store all of the } v_1, v_2, \ldots. \text{ Since there are only 16 possibilities you can do this in an inelegant but easy way. And DO NOT WORRY- I am NOT going to ask you to later to it for 10 bits or 100 bits or something that would require a clever way to store it.)}
\]

(b) For each \( (b_3, b_2, b_1, b_0) \) output the length of the sequence before a repeat. sequence without a repeat.

You final output should look like this (I made up the numbers and only gave the first two rows. Yours should have 16 rows.)

<table>
<thead>
<tr>
<th>( b )-vector</th>
<th>length of sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
<td>5</td>
</tr>
<tr>
<td>0001</td>
<td>19</td>
</tr>
</tbody>
</table>

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4. (30 points) Give a rigorous definition of a pseudorandom FUNCTION that uses a game. It should begin with $F_k(x)$ where $k$ is unif in $\{0, 1\}^n$. $F_k$ goes from $\{0, 1\}^n$ to $\{0, 1\}^n$.

(ADDED HINT:

- Use a Game!
- In class we have defined roughly two kinds of games: Here is an example of each: (1) Defining perfect security. Here Eve picks $m_0, m_1$, Alice encodes one of them into $c$, gives Eve $c$, and eve has to tell which one it was $m_0$ or $m_1$. (2) Defining Psuedorandom GEN: Here ALICE picks a truly random string and a pseudorandom string and Eve has, gives one of them to Eve, Eve has to tell which one it was.
- The definition of Psuedorandom Function will be more like that of pseudorandom generator.
- When Alice gives Eve a function, Eve will have black box access to it.)