1. (0 points) What is your name? Write it clearly. What day is the midterm? Staple your HW.

2. In this problem the alphabet is \{a, b\}.
   
   (a) (20 points) Write a regular expression for \{w : n_a(w) \equiv 0 \pmod{3}\}.
   
   (b) (20 points) Write a regular expression for \{w : n_a(w) \equiv 1 \pmod{3}\}.

SOLUTION TO PROBLEM 2
a) \(b^*\left(b^*ab^*ab^*ab^*\right)b^*\).

b) \(b^*ab^*\left(b^*ab^*ab^*ab^*\right)b^*\).

END OF SOLUTION TO PROBLEM 2

3. Alphabet is \{a\}.

   (a) (10 points) Write a DFA for the language \(\{a^i : i \neq 1000\}\) (you may use ...). How many states does it have?

   (b) (10 points) Let \(n \in \mathbb{N}\). Think of \(n\) as large. Write a DFA for the language \(\{a^i : i \neq n\}\) (you may use ...). How many states does it have (this will be a function of \(n\)).

   (c) (0 points but please think about – Please do so by the REAL day its due, March 6 so we can discuss in class) The answer to the last part was roughly \(n\) (for example, it might be \(n + 1\)). Is the following true or false: Any NFA for \(L\) requires around \(n\) states. Try to prove or disprove.

SOLUTION TO PROBLEM 3
Omitted

END OF SOLUTION TO PROBLEM 3

TURN THE PAGE
4. In this problem we go through a VERY simple case of going from a DFA to a regular expressio. DO NOT CHEAT- follow the construction. The alphabet is \{a, b\}.

(a) (10 points) Write an NFA for the language 

\[ L = \{w : \text{a is the first letter of } w \} \]

that has only two states. Label the two states 1 and 2 where 1 is the start state and 2 is the other state (which is the only final state).

(b) (27 points) Compute, in order, and using the algorithm show in class. Show all steps.

\[
\begin{align*}
R(1, 1, 0) \\
R(1, 2, 0) \\
R(2, 1, 0) \\
R(2, 2, 0) \\
R(1, 1, 1) \\
R(1, 2, 1) \\
R(2, 1, 1) \\
R(2, 2, 1) \\
R(1, 2, 2) \text{ (this is the only one I need)}
\end{align*}
\]

SHORT CUTS YOU CAN USE (You can use other ones also but be careful)

For any reg exp \(\alpha\), \(\emptyset \cdot \alpha = \emptyset\).

If \(\sigma \in \{a, b\}\) then \((\sigma \cup e)^* = \sigma^*\).

\[e^* = e\]

For any regular expression \(\alpha\), \(e\alpha = \alpha\) and \(\alpha e = \alpha\).

For any \(\sigma \in \Sigma\), \(a \cup a = a\).

(c) (3 points) From your work on part 1 write down a regular expressio for \(L\). (NOTE- it should be longer than the obvious reg exp for \(L\) which is \(a(a \cup b)^*\).)
SOLUTION TO PROBLEM 4

a)
\[ \delta(1, a) = 2 \]
\[ \delta(1, b) = \emptyset \]
\[ \delta(2, a) = 2 \]
\[ \delta(2, b) = 2. \]

b)
\[ R(1, 1, 0) = e \]
\[ R(1, 2, 0) = a \]
\[ R(2, 1, 0) = \emptyset \]
\[ R(2, 2, 0) = b \cup e \]

\[ R(1, 1, 1) = R(1, 1, 0) \cup R(1, 1, 0) R(1, 1, 0)^* R(1, 1, 0) = e \cup ee^* e = e \]
\[ R(1, 2, 1) = R(1, 2, 0) \cup R(1, 1, 0) R(1, 1, 0)^* R(1, 2, 0) = a \cup ee^* a = a \]
\[ R(2, 1, 1) = R(2, 1, 0) \cup R(2, 1, 0)(R(1, 1, 0)^* R(1, 1, 0) = \emptyset \cup \emptyset = \emptyset . \]
\[ R(2, 2, 1) = R(2, 2, 0) \cup R(2, 1, 0) R(1, 1, 0)^* R(1, 2, 0) = (b \cup b) \cup \emptyset = b \cup e \]

\[ R(1, 2, 2) = R(1, 2, 1) \cup R(1, 2, 1) R(2, 2, 1)^* R(2, 2, 1) = a \cup a (b \cup e) (b \cup e) = ab^* b^* \]

END OF SOLUTION TO PROBLEM 4