1. (0 points) What is your name? Write it clearly. Staple your HW. When is the midterm? Where is the midterm?

AN INJECTION IS ALSO CALLED A 1-1 MAPPING.

2. (25 points) Prove that if there is an injection from $A$ to $B$ and an injection from $B$ to $A$ then there is a bijection from $A$ to $B$ (this is called the Cantor-Schroder-Bernstein by some and the Schroder-Bernstein theorem by others, and likely other combinations by other people. You MAY go to the web and find a proof; however, when you write it up put it in your own words and make sure you understand it.) You may use this result throughout the HW.

3. (25 points)

(a) Show there is an injection from $\{0, 1\}^\omega$ to $\{0, 1, 2\}^\omega$ (HINT: this is trivial).

(b) Show there is an injection from $\{0, 1, 2\}^\omega$ to $\{0, 1\}^\omega$

(c) From the two above statements what can you conclude?

SOLUTION TO PROBLEM 3

1) The map $f(x) = x$ is an injection.

2) I first say how to map must the symbols 0,1,2. Map 0 to 00, 1 to 11, and 2 to 01 Now just concat. So for example

$f(01120) = 0011110100$

From the output you can recover the input so its an injection. For example, lets say you were told that

$f(x) = 1101110100$ and asked what the input must have been. You know!

We rewrite the output with spaces for clarity. It is

11 01 11 01 00
Hence $x = 12120$.

3) There is an injection both directions, so by the previous problem we know there is a bijection from $\{0, 1, 2\}^\omega$ to $\{0, 1\}^\omega$.

END OF SOLUTION TO PROBLEM 3

4. (25 points) Let PRIMES be the set of primes. Show that the set of all functions from $\mathbb{N}$ to PRIMES is uncountable.

SOLUTION TO PROBLEM FOUR
Assume, by way of contradiction, that the set of such functions is countable. So $f_1, f_2, f_3, \ldots$ is the set of all function from $\mathbb{N}$ to PRIMES.
We construct a function NOT on that list $g(x) = \text{the next prime after } f_x(x)$.
For all $i$, $g(i) \neq f_i(x)$, hence $g$ is not $f_i$. Hence $g$ is not on the list.

END OF SOLUTION TO PROBLEM FOUR

5. (25 points) Let the set Josh be defined as follows:

- $(\mathbb{Z}[x]$ is the set of polynomials in one variable $x$ with coefficients in the $\mathbb{Z}$ which is the integers$.)$ If $p(x) \in \mathbb{Z}[x]$ and $\alpha$ is any of the transcendental Numbers listed on the website of 15 awesome transcendental numbers (there is a pointer on the course website) then $p(\alpha)$ is in Josh.

- If $p$ is a polynomial with integer coefficients and $n \in \mathbb{N}$, $n \geq 2$, then $p(ln \ n)$ is in Josh.

Is Josh countable or uncountable? Justify your answer.

SOLUTION TO PROBLEM FIVE
Countable.
$\mathbb{Z}[x]$ is countable (we showed this in class while showing that the Algebraic Numbers are countable). We list them out

$$p_1, p_2, \ldots,$$

We define sets $A_1, A_2, \ldots$
$A_i = \{ p(\alpha) : \alpha \text{ is one of the 15 awesome Trans Numbers } \} \cup \{ p(\ln n : n \in \mathbb{N}, n \geq 2) \}$

Each $A_i$ is the union of a finite set and a countable set, hence its countable.

Josh is the union of all of the $A_i$’s and hence is countable.

END OF SOLUTION TO PROBLEM FIVE